MODELING MULTIPLE NONLINEAR TIME SERIES: A GRAPHICAL APPROACH TO THE TRANSFER FUNCTION

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KEY WORDS: SETAR time series, Ecological time series, Case study

ABSTRACT

A transfer function model for the interrelationship of multiple nonlinear time series is developed for the case of an explanatory nonlinear time series. Directed graphs and cross-directed-graphs are used to explore the common trajectory of the interrelated time series.

A Case Study of the monthly U.S. Stroke mortality, U. S. Respiratory Mortality, and the monthly mean U.S. temperature time series from 1938 to 1989. Influenza epidemics may affect stroke mortality, since the stroke patients form a pool of susceptibles that will have stroke as the primary cause of death. Elevated summer temperatures have been hypothesized as accelerating stroke mortality.

INTRODUCTION

The detection of nonlinearity in a univariate time series can be performed

- graphically with directed scattergrams and directed multigrams. A directed graph plots each point versus the previous point in time and collects the points in order of occurrence. A cross-directed-graph follows the trajectory of the two different time series over time. The subsequence directed graph partitions the series up into different segments to test whether the character of the series evolves over time.
- on a global basis with tests based on comparing the autocorrelation properties of the residuals squared with the autocorrelation properties of the squared residuals (Box-Pierce and Ljung-Box Portmanteau tests). The residuals from the seasonal effects are used to test for nonlinearity of the transfer function between the two time series (Brockwell). The residuals from the ARIMA fitting of Y_t on itself, can be used directly to estimate the transfer function (Enders) provided the non-X component is linear.
- with tests based on the homogeneity of the series over time (runs tests or chi-square tests of heterogeneity).

This paper takes the problem of identification of nonlinearity of multiple time series through a transfer function approach. Each series is tested for nonlinearity itself, and if nonlinearity is present, then the relationship between the two series is tested for linearity. Both graphical and quantitative methods for identification of nonlinearity will be explored.

THE MODEL

A linear time series is usually expressed in terms of an autoregressive and a moving average component (Box and Jenkins). The autoregressive component, e.g., a first order autoregressive series, AR(α), can be written as

$$X_t = \alpha X_{t-1} + \varepsilon_{t-1}$$

The simplest nonlinear time series is a segmented time series which switches between two different AR(α) processes on a regular or "seasonal" basis. The next step in complexity is the SETAR(α_1, α_2 ; τ) which switches from one AR(α) to another whenever the observed value of the series passes a threshold τ.

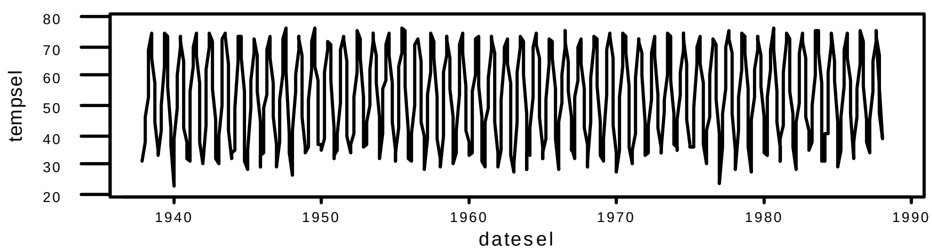
The transfer function model assumes that there is an underlying nonlinear time series

$$X_t = \alpha_1 X_{t-1} + \varepsilon_{t-1} \text{ if } X_t > \tau.$$

$$X_t = \alpha_2 X_{t-1} + \varepsilon_{t-1} \text{ if } X_t \le \tau.$$

and that the outcome series, Y_t, depends on X_t through a linear or nonlinear function F(.)., as well as on previous values of Y_t. The non-X portion of Y can also be nonlinear; however, for this model it will be assumed that the nonlinearity is induced through the covariate series X_t. Consequently the model for Y_t is

$$Y_t = \beta Y_{t-1} + F(X_t) + \xi_t$$



80
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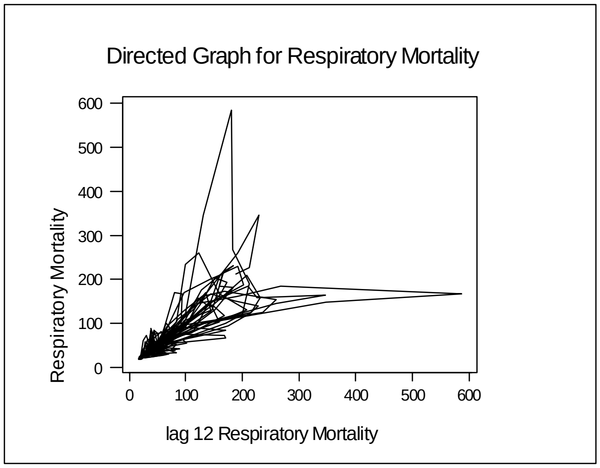
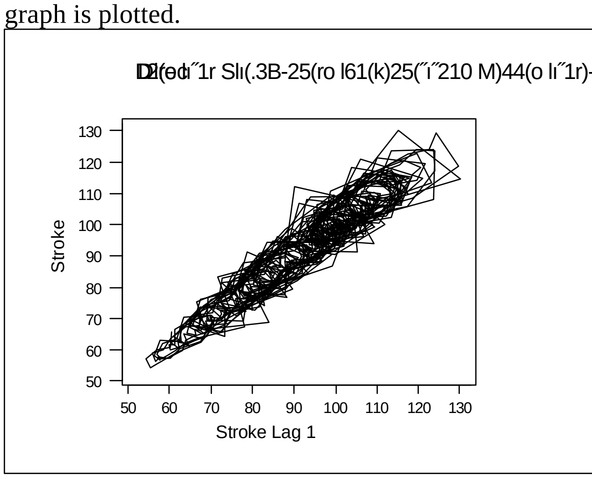
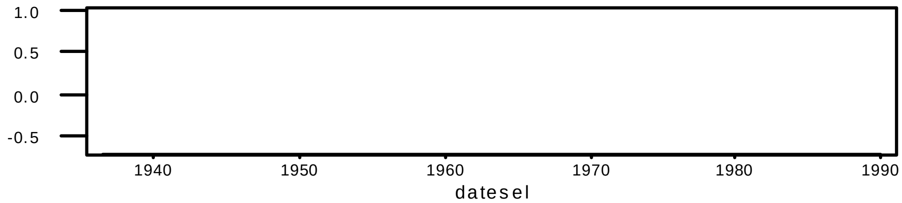


Figure 4 shows a directed graph of the lag 1 stroke mortality; the yearly (lag 12) curve looks essentially the same because of the time trend. Stroke mortality needs to be detrended before the directed graph is plotted.




1.0
0.5
0.0
-0.5
1940
1950
1960
1970
1980
1990
datesel

An alternative method for removing the seasonality is to use a sin and cosine to represent the seasonality, and to fit this to respiratory and stroke mortality separately. In this case the cross-correlation between the two curves is 0.575. The cross-correlation between the ARIMA residual series was 0.433.